

## Structures Multi-Damage Detection by Discrete Wavelet Entropy Method

Milad Ashurzade Moghri<sup>1</sup> Ali Bigleri Fadafan<sup>2</sup>

### 1. Introduction

Identification of infrastructure damages in early stages is one of the most fundamental requirements in the maintenance process. In order to identify damages in the process of Structural Health Monitoring, numerous methods have been developed, including Neural Network (NN) Damage Identification Techniques, Time Series Damage Identification Techniques, Frequency Response Damage Identification Techniques, Force Spectrum Density Damage Identification Techniques, and Wavelet Damage Identification Techniques.

In this study, damage detecting method in the structure is proposed based on the concept of wavelet entropy, using discrete wavelet data analysis. Then, the proposed method for identifying damaged points in seismic analysis process of 3D structure was evaluated.

### 2. Discrete relative wavelet entropy in damage identification

Complex signals analysis by an acceptable rate is one of the discrete wavelets transform properties. In order to utilize discrete wavelet transform properties, it is necessary to increase the accuracy of identification function leading to an increase in the quality of damage detection. In discrete wavelet transform, the input signal is analyzed by considering the mother wavelet function, while input signal is decomposed to low frequency  $LF(k)$  and high frequency  $HF(k)$  signals, using low and high frequency filters. These filters are designed to extract data in the predefined range to precisely control the process at different scales.

$$\Psi_{(t)}^{2i} = \sqrt{2} \sum_{k=-\infty}^{\infty} HF(k)\Psi^i(2t - k) ; \quad (1)$$

$$\Psi_{(t)}^{2i+1} = \sqrt{2} \sum_{k=-\infty}^{\infty} LF(k)\Psi^i(2t - k) ; \quad (2)$$

By applying  $n$  discrete wavelet scale, discrete wavelet coefficient signal is calculated for the last approximation ( $A_n$ ) and other subgroups ( $D_i; i = n, n - 1, \dots, 1$ ). Each of these scales can be considered a separate subgroup of different lengths. The result structure is presented in Figure 1 for the scale 3.

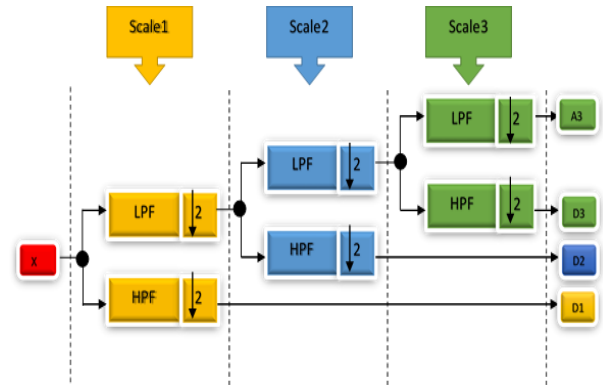


Figure 1. The computational structure of discrete wavelet transfer

When the main vibration signal is defined by  $L$  components,  $A_n$  composed of  $\frac{L}{2^n}$  and ( $D_i; i = n, n - 1, \dots, 1$ ) composed of  $\frac{L}{2^i}$  components. The vector of discretized wave components can be arranged as a matrix of scaled subgroups, respectively, increasing the frequencies as  $C_{ji} = [A_n, D_n, D_{n-1}, \dots, D_1]$  for the scale  $n$ . Each of the  $C_{ji}$  components vectors has  $k$  arrays.

The signal energy for each subgroup, shown with  $E_j$ , is defined as squares of the components summation. It can be calculated for the entire length of the subgroups. This energy can be written as follows for the  $k$  components,

$$E = [E_j] = \left[ \sum_k |A_n(k)|^2, \sum_k |D_j(k)|^2 \right] \quad (3)$$

and total energy of wavelet shown by  $E_{tot}$  defined as the summation of the energy vector components,

$$E_{tot} = \sum_k |A_n(k)|^2 + \sum_{i=1}^n \sum_k |D_j(k)|^2 \quad (4)$$

The wavelet energy ratio for the  $j$  subgroup, represented by  $P_j$ , can be defined as equation (5).

$$P_j = \frac{E_j}{E_{tot}} \quad j = 1, 2, \dots, n \quad (5)$$

where  $P_j$  is the wavelet energy percentage for the  $j$  subgroup. The distribution of  $P_1, P_2, \dots, P_n$  values is somewhat similar to an energy probability distribution, meaning that the summation of values is equal to one.

It can be shown that using wavelet entropy ratio, equation (6), for damaged and undamaged structure,

<sup>1</sup>. MSc. Student in Civil-Structural Engineering, Civil Engineering Department, Golestan University, Golestan, Iran

<sup>2</sup>. Corresponding author. Assistant Professor, Civil Engineering Department, Golestan University, Golestan, Iran.  
Email: biglari.a@gmail.com

$P_j^{Damaged}$ ,  $P_j^{Undamaged}$ , improves the accuracy of damage identification,

$$RWE = - \sum_j P_j^{Damaged} \ln\left(\frac{P_j^{Damaged}}{P_j^{Undamaged}}\right) \quad (6)$$

For an undamaged structure, the wavelet energy for damaged will be equal to undamaged condition,  $P_j^{Damaged} = P_j^{Undamaged}$ . Therefore, the RWE value will be zero, but a damage will change the response signal, and as a result, the RWE value will increase. The computational algorithm is displayed in Table 1.

### 3. Conclusion

In this study, a method for identifying damages based on discrete wavelet entropy was proposed and evaluated. In order to show efficiency of the introduced damage index for this wavelet, efficiency of the proposed algorithm in the 3D construct model was explored. Then, the possibility of using this algorithm in some damage scenarios were investigated and damages were correctly identified.

The results of using this identification method can be summarized as follows:

1. The proposed algorithm and the indicator have ability to accurately identify location of damage;
2. In each individual and multiple samples of damage, all damaged degrees can be successfully identified in each damage scenarios;
3. The proposed entropy index in healthy freedom degrees is not affected by damage in adjacent freedom degrees, but changes can be a good guide to the next stages of evaluation.

It should be noted that one of the advantages of discrete wavelet conversion is less computational effort than Fast Fourier Transform (FFT). Furthermore, applying the concept of wavelet entropy in discrete wavelet transforming process will improve the accuracy of damage detection process. These capabilities have a very effective role in practical process of damage assessment in a structure.

Table1. Computational algorithm

Start	
Extract node Acceleration for damaged and undamaged structures	
Repetition of this loop for undamaged structure and damaged structure	
Loop repetition for all acceleration in the examined degrees of freedom	
<b>Start Damage Identification</b>	
Loop repetition on the scale examined for $n$ scales 1, 2, ..., $n$	
Calculation of vector components $A_n$	
Calculation of vector components $D_1, D_2, \dots, D_n$	
Calculation of energy functions $E_1, E_2, \dots, E_n$	
Calculation of total energy function $E_{tot}$	
Calculation of energy ratio $P_1, P_2, \dots, P_n$	
Calculation of damage function in all degrees $RWE$	
Identify maximum damage functions in checked degrees of freedom and detect damaged degrees of freedom	
the end	