Journal of Ferdowsi Civil Engineering, Vol.33, No.1, 2020.

Isogeometric Shape Optimization of 2D Structures using Fully Analytical Sensitivity Analysis

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1. Introduction

Since the resources available to human beings in nature are limited, it is very important to use these resources as efficiently as possible. In this respect, optimization which can be simply defined as the process of searching for the best, serves as a valuable tool. The main purpose of engineering design is to find the best possible solution to a specific problem, optimization is therefore at the very heart of engineering.

In this regard, significant progress has been made in the last three decades in the field of the shape optimization of the structures. Shape optimization is a process in which the optimal shape of a structure using a repetitive process based on structural response analysis and sensitivity calculation is obtained, which is raised in the Gradient-Based optimization branch.

In most of the structural optimization approaches analysis and sensitivities are carried out by using the conventional finite element method. Due to the variations of the problem geometry within the shape optimization iterations, updating the computational model by several remeshings is required that is quite time consuming and computationally costly. As a remedy for this problem, the NURBS-based isogeometric analysis method was adopted that combines the Computer Aided Design (CAD) and finite element techniques.

In this study, in the shape optimization of the structures, the Method of Moving Asymptotes (MMA), which is gradient-based, was employed and to calculate the required sensitivities a fully analytical approach is presented. To demonstrate the performance of the suggested approach, various numerical examples are solved.

2. NURBS Function

This section is devoted to the introduction of NURBS functions and surfaces, followed by a brief description of the method of isogeometric analysis in two-dimensional problems. The NURBS surface is obtained from the linear combination of the basic functions and the control points as follows

$$S(\eta,\zeta)=\sum_{i=1}^{n}\sum_{j=1}^{m}R_{i,j}^{p,q}(\eta,\zeta)P_{i,j} \tag{1}$$

3. Shape Optimization

The purpose of shape optimization of the structure is to find the geometry of the boundaries of the structure so that the specific behavior of the structure is in the best condition. Usually, the constraints could be geometrical such as restrictions on the width or height of the structure or behavioral such as restrictions on stresses, displacements, and natural frequencies. According to these explanations, an optimization problem can be represented as follows

$$\begin{cases}\text{Objective Function:}\quad min\ \ \Im(\boldsymbol{x},\mathbf{q}(\boldsymbol{x}))\\ \text{Constraints:}\begin{cases}h_l(\boldsymbol{x},\mathbf{q}(\boldsymbol{x}))=0, & l=1,\dots,\\ g_k(\boldsymbol{x},\mathbf{q}(\boldsymbol{x}))\le 0, & k=1,\dots,\\ x_j^{min}\le x_j\le x_j^{max}, & j=1,\dots,\end{cases}\end{cases} \tag{2}$$

4. Sensitivity Analysis

Sensitivity analysis means examining the degree of dependence of the design on each influential parameter. In most of the mathematical programming algorithms, to find a search direction, sensitivity information is required. The derivatives of the objective and constraint functions with respect to the design variables are calculated using a sensitivity analysis, which indicates the sensitivities of the current design to small changes in the design variables. The sensitivities can be computed by finite difference, analytical or semi-analytical methods. In the analytical methods used in this study, the values of the derivatives from explicit functions are obtained according to the problem design variables.

5. Results

To show the accuracy and efficiency of the proposed method, an example is given to the shape optimization of the inner hole boundaries of the plate with plane stress conditions. This plate is subjected to biaxial stresses $\sigma_x=\sigma_y=2.5\,\mathrm{kN.m^{-1}}$ (Figure 1). The dimensions of the structure discussed in this example are selected as $x=75\ \mathrm{m}$ and $y=25\ \mathrm{m}$, respectively. The material parameters are the Young's modulus $E=210\times10^3\mathrm{GPa}$ and the Poisson ratio $\nu=0.3$. This plate is subjected to volume constraint $V\le\hat{V}$, in which $\hat{V}$ is the equation of 99% of the initial volume of the whole plate. The lower bound, upper bound, and optimal values of the design

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variables are shown in Table 1. It is clear that the shape of the hole boundary is close to the geometry of the circle.

6. Conclusion

In this paper, the shape optimization of the two-dimensional structures using the isogeometric analysis method based on NURBS basic functions was developed. One of the most important advantages of this method over conventional methods, which use the classical finite element method, is the elimination of the finite element mesh construction phase during the optimization process, which is repeated at all steps. This case significantly reduces the volume and cost of calculations. In this paper, the analytical formulation is also used to calculate the sensitivity, which is used in gradient-based optimization algorithms, in shape optimization using the isogeometric method. The analytical method in this study has been used only for two-dimensional problems, but it can also be used for three-dimensional problems.

Table 1. Variation of design variables in the plate with hole

Design variable	Lower bound	Upper bound	Initial value	Optimal value
x_1	-50	0	-25	-22.489
x_2	-50	0	-18.75	-22.479
x_3	-50	0	-6.25	-8.987
y_2	0	50	6.25	8.999
y_3	0	50	18.75	22.472
y_4	0	50	25	22.483

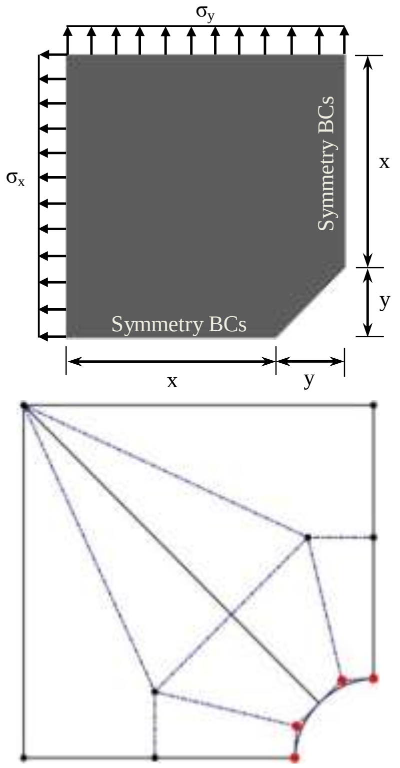

Figure 1. Shape optimization of the plate with hole