Journal of Ferdowsi Civil Engineering, Vol.32, No.2, 2019.

Elastic Analysis of Multi-Layer Plates with Arbitrary Geometry and Boundary Conditions Using Layerwise Mixed Formulation

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1. Introduction

Due to the anisotropy behavior, composite materials have some distinctive features in comparison to the isotropic materials. Typically, composite materials have a high hardness and strength in the direction of the fibers. By combining several composite layers, a composite plate with the desired properties can be obtained. Due to the increasing use of composite materials, it is necessary to provide more accurate methods for investigating the behavior of these structures in order to optimize them.

The variety of theories for analysis of multi-layer composite plates is mainly due to the assumptions related to the variation of unknown fields along the thickness of the plate. The solutions based on the classical plate theory (CPT), correspond well with the exact 3D elasticity solutions for thin plates, while the accuracy of results based on of this theory is lost for thick plates. In this regard, shear deformation, zigzag, and layerwise theories and 3D elasticity-based methods have been developed for the analysis of thick plates. In analyzing multi-layer composite plates, it is important to obtain more accurately the interlaminar stresses. To calculate more accurately interlaminar stresses, mixed formulations are established. In this paper a partial mixed formulation based on the layerwise theory is presented for multi-layer composite plates with arbitrary geometry. In the presented method, essential as well as natural boundary conditions are satisfied exactly. Moreover, the continuity of interlayer traction is satisfied exactly.

2. Mixed layerwise formulation

Consider a composite plate with arbitrary geometry consisting of M layers $\Omega_i, i=1,2,\dots,M$, as shown in Figure 1.

The Cartesian coordinates system is chosen so that the z coordinate be aligned with the thickness of the layers. The displacement functions u, v, w as well as the out-of plane stresses $\sigma_{xz}, \sigma_{yz,}\sigma_{zz}$ are expressed in the kth layer, based on the mixed layerwise theory, as follows

For $\alpha = u, v, w, \sigma_{xz}, \sigma_{yz,}\sigma_{zz}$,

$$\alpha^{(k)} = \sum_{n=0}^{N_\alpha} \varphi^{(k)}_{n-(\alpha)}(x,y)\,\psi^{(k)}_{n-(\alpha)}(z)\ , \tag{1}$$

$$\varphi^{(k)}_{n-(\alpha)}(x,y) = B_\alpha(x,y)\sum_{m=0}^{p_\alpha}\sum_{l=0}^{p_\alpha-m} d_r^{(\alpha)-\gamma}x^m y^l\ , \tag{2}$$

$$\psi^{(k)}_{n-(\alpha)}(z) = \prod_{\substack{i=0\\ i\neq n}}^{N_\alpha} \frac{\left(z-z_i^{(k)}\right)}{\left(z_i^{(k)}-z_i^{(k)}\right)}\ , \tag{3}$$

in which $B_\alpha(x,y)=1, \alpha=\sigma_{xz}, \sigma_{yz,}\sigma_{zz}$ and $B_\alpha(x,y), \alpha = u, v, w$ are selected for enforcing essential boundary conditions. p_α and N_α are the order of polynomials, r and γ are the counters, and $d_r^{(\alpha)-\gamma}$ are the unknown coefficients.

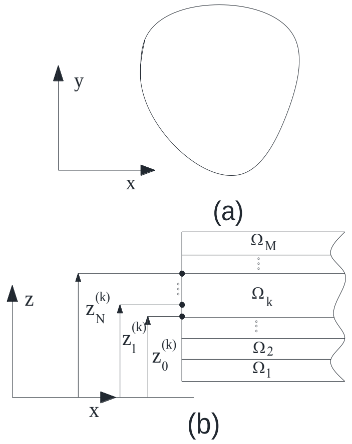

Figure 1. A multi-layer composite plate with arbitrary geometry: top view; (b) front view

Equilibrium and compatibility equations are also applied with desired accuracy, using the Reissner's variational principle as

$$\sum_{k=1}^{M}\iiint_{\Omega_k}\left[\delta\boldsymbol{\varepsilon}_{pG}^T\boldsymbol{\sigma}_{pH} + \delta\boldsymbol{\varepsilon}_{nG}^T\boldsymbol{\sigma}_{nM} + \delta\boldsymbol{\sigma}_{nM}^T(\boldsymbol{\varepsilon}_{nG} - \boldsymbol{\varepsilon}_{nH})\right] d\Omega_k - \delta W^e = 0\ , \tag{4}$$

in which $\boldsymbol{\sigma}_{nM}$ are the out-of plane stresses defined in Eq. (1), $\boldsymbol{\varepsilon}_{pG}$ are the in-plane strains obtained from strain-displacement relations. Subscript H indicates that the quantity is obtained from the Hook's law. Eq. (4) leads to a set of linear algebraic equations in terms of unknown coefficients, d.

3. Results and Discussion

For verification and examination of the efficiency of the method, two cross ply composite plates, three-layer $0°/90°/0°$ and four-layer $0°/90°/90°/0°$ composite plates under transverse sinusoidal loading are considered. The thickness of each layer is assumed to be h. Assuming the fibers are aligned in the x direction, the elastic properties of each layer are

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$$E_1 = 25E_2, E_2 = E_3,\ G_{12} = G_{13} = 0.5\ E_2,\ \ G_{23} = 0.2\ E_2,\ \ \nu_{12} = \nu_{13} = \nu_{23} = 0.25\ , \qquad (5)$$

Variations of normalized out-of plane stresses, $\overline{\sigma}_{xz}(a, a/2, z)$, $\overline{\sigma}_{yz}(a/2, a, z)$, and $\overline{\sigma}_{xx}(a/2, a/2, z)$ along the thickness are plotted in Figures 2 to 4. In the figures, trend of convergence of displacement and mixed layerwise theories is shown and compared with the exact solution.

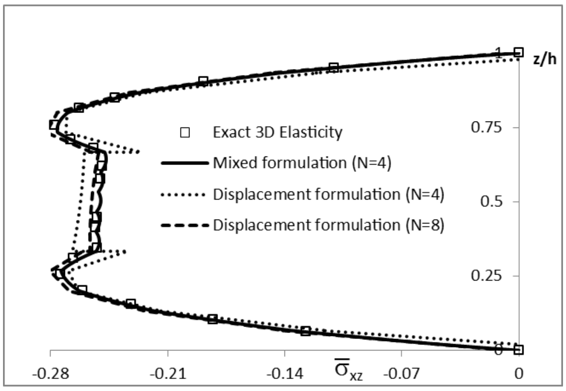

Figure 2. Variation of the shear stress $\overline{\sigma}_{xz}(a, a/2, z)$ through the thickness for 0° / 90° / 0° composite plate

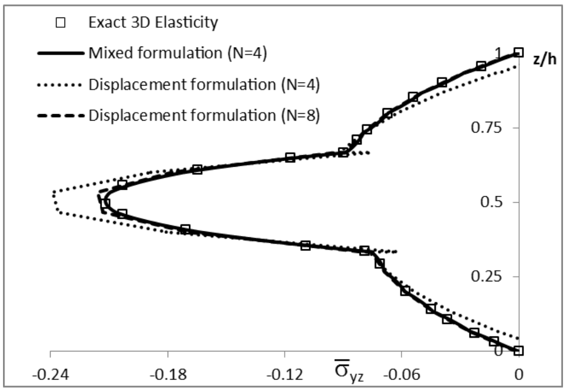

Figure 3. Variation of the shear stress $\overline{\sigma}_{yz}(a/2, a, z)$ through the thickness for 0° / 90° / 0° composite plate

These figures show that for $N = 4$, results of mixed formulation are better in agreement with the exact solution than those of displacement formulation. In the case of $N = 4$, in displacement formulation the traction shear stresses $\overline{\sigma}_{xz}(a, a/2, z)$ and $\overline{\sigma}_{yz}(a/2, a, z)$ are discontinuous and are not in agreement with the exact solution while in mixed formulation those are continuous and are in good agreement with the exact solution. As seen in Figures 2-4, the accuracy of the results of displacement formulation is improved when N is increased from 4 to 8.

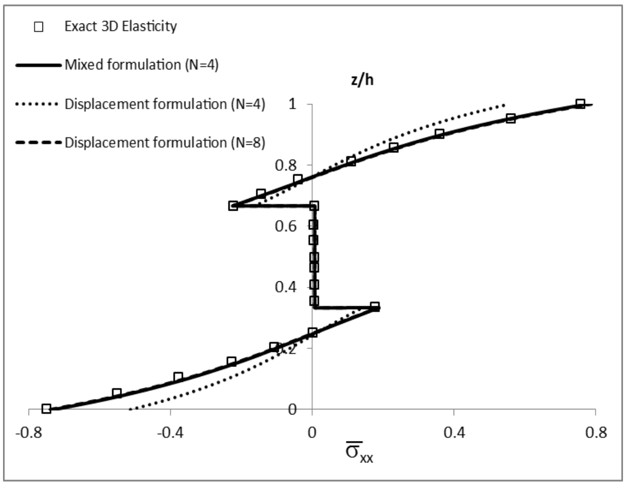

Figure 4. Variation of the normal stress $\overline{\sigma}_{xx}(a/2, a/2, z)$ through the thickness for 0° / 90° / 0° composite plate

4. Conclusion

In this paper, a three-dimensional semi-analytical solution with mixed layerwise formulation is presented for multi-layer composite plates with arbitrary geometry and boundary conditions. In this study, three components of the displacement field and three components of the out-of plane stresses are considered as a sum of a series of functions with unknown coefficients. These functions are chosen so that the essential homogeneous boundary conditions and the non-homogeneous natural boundary conditions are exactly satisfied. Also, continuity of the displacement field and the traction stresses between the layers in the boundary between the adjacent layers is exactly satisfied. Equilibrium and compatibility equations are also applied with desired accuracy, using the Reissner's variational principle. The results show that the mixed formulation has faster convergence than displacement based formulation, and provides more accurate values of interlaminar stresses. Out of plane stresses obtained by this method are in very good agreement with those of the exact solution.