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Sensitivity Analysis of Fuzzy Structural Reliability Using Adaptive Stability Transformation Method

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1. Introduction

A typical structural reliability analysis deals with mathematical models which describe the structural geometry, loads, and material properties. These models relate the resistance and a load of structural components. The function namely the Limit State Function or performance function is a combination of random variables. In the limit state function, there are parameters which do not possess constant values and usually are of random and vague identity in nature. Hence, there is usually a sort of uncertainty in the safety assessment of structures.

The nature of uncertainties and the modeling methods in the reliability analysis of structures have been of researchers' interest for many years. Recently various approaches have been proposed to handle epistemic uncertainty of random variables in the probabilistic models using fuzzy sets. From among optimization approaches are more popular due to their applicability in most engineering problems. Moreover, the determination of fuzzy reliability index requires a robust reliability method. In this paper, modeling of epistemic uncertainty of random variables in the reliability analysis process has been done based on two elitist genetic optimization approach and first-order reliability method based on adaptive stability transformation method (ASTM) aiming to determine fuzzy reliability index. Survey results indicate that the proposed method more robust to determine the fuzzy reliability index and is able to perform sensitivity analysis regarding the epistemic uncertainty.

2. Proposed fuzzy reliability analysis procedure

In the proposed method, after determining the same alpha cuts α_k of fuzzy random variables, a hyperspace namely crisp subspace X_{α_k} is constructed. If the fuzzy reliability problem consists of two, three or n fuzzy random variables, the crisp subspace is in the form of a rectangle, cube, or an n-dimensional hyper cubic, respectively.

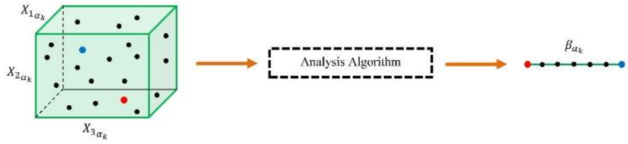

Fig. 1. Crisp subspace X_{α_k} and β_{α_k} interval

As shown in Figure 1, points inside the crisp subspace are used as inputs of analysis algorithm to determine the relative interval of fuzzy reliability index β_{α_k}. Assembling these alpha cuts, yields the membership function of fuzzy reliability index. However, there are two optimums to in the crisp subspace which result in the corner of β_{α_k} set.

So determination of minimum and maximum fuzzy reliability index interval entails to the search for two optimums in the crisp subspace. The schematic flowchart of the proposed method is depicted in Figure 2.

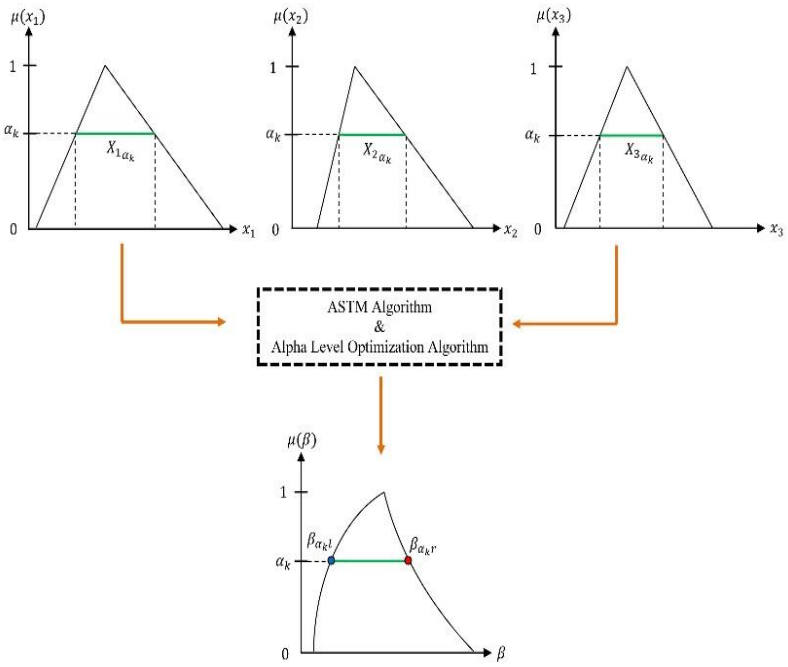

Fig. 2. Schematic flowchart of the proposed method

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3. Adaptive stability transformation reliability method

In order to calculate fuzzy reliability index, an efficient and robust reliability method that could calculate the reliability index (β) for different statistical properties of random variables according to the following equation, must be applied.

$$P_f = \int_{g(\boldsymbol{X})\le 0} \cdots \int f_{\boldsymbol{X}}(x_1,\ldots,x_n)dx_1..dx_n \approx \Phi(-\beta) \qquad (1)$$

Where, $g(\boldsymbol{X})$ is the limit state function and $f_{\boldsymbol{X}}$ is joint the probability density function of random variables X and Φ is the cumulative normal standard probability density function. Generally, most probable point (MPP) could be determined using the largest decent in the form of following nonlinear mapping:

$$f(\boldsymbol{U}) = \frac{\nabla^T g(\boldsymbol{U}_k)\boldsymbol{U}_k - g(\boldsymbol{U}_k)}{\nabla^T g(\boldsymbol{U}_k)\alpha_k}\alpha_k \qquad (2)$$

In which, α is the normalized steepest descent vector at $\boldsymbol{U}_k$ the point. The stability transformation method could be applied as a suitable method to control the instability of the first-order reliability method, in the form of the following nonlinear discrete mapping:

$$\boldsymbol{U}_{k+1} = \boldsymbol{U}_k + \lambda_k[f(\boldsymbol{U}_k) - \boldsymbol{U}_k] \qquad (3)$$

Where, λ_k is the stable control coefficient in the iteration k-th, which is determined according to an adaptive control function according to Eq. (4).

$$f_C(k) = \|f(\boldsymbol{U}_k) - \boldsymbol{U}_k\| \qquad (4)$$

Similarly, it could be concluded that $f_C(k-1) = \|\boldsymbol{U}_k - \boldsymbol{U}_{k-1}\|$. Control function $f_C(k)$ may result in a successful iteration for stability transformation while $f_C(k) < f_C(k-1)$ (that's mean: $\lim_{k\to\infty} f_C(k) \approx 0$). So it can be concluded that $\boldsymbol{U}_{k+1} \approx \boldsymbol{U}_k$. It can be conducted that the new formulation of the FORM for fuzzy reliability analysis can be provided stable results to achieve the stabilization of the reliability index at each iteration.

4. Example: Nonlinear dynamic system

Here a nonlinear mass-spring system under the rectangular pulse is considered. The limit state function is defined according to Eq. (5)

$$g = 3r - \left|\frac{2F}{mW^2}\sin\left(\frac{Wt_1}{2}\right)\right| \qquad (5)$$
$$W^2 = \frac{C_1 + C_2}{m}$$

Implementation of the proposed method yields the membership function of fuzzy reliability index according to Figure 3. In the analysis process, the gradient function has been estimated for 48,868 times including 635,284 limit state call function during the 2,903 seconds CPU-run time. Sensitivity analysis of fuzzy reliability index due to epistemic uncertainty using Shannon-entropy measure results for $H_u(\tilde{\beta}) = 0.807k$, which proves that the reliability index is more sensitive to epistemic uncertainty. Also, in order to examine the sensitivity of fuzzy reliability index to the uncertainty interval of mean and standard deviation of random variables, results for different values of interval is shown in Figure 4.

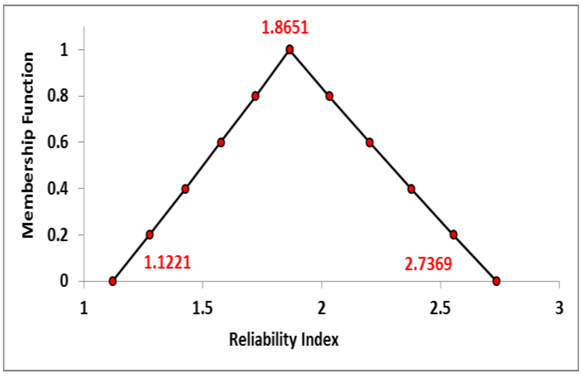

Fig. 3. Fuzzy reliability index

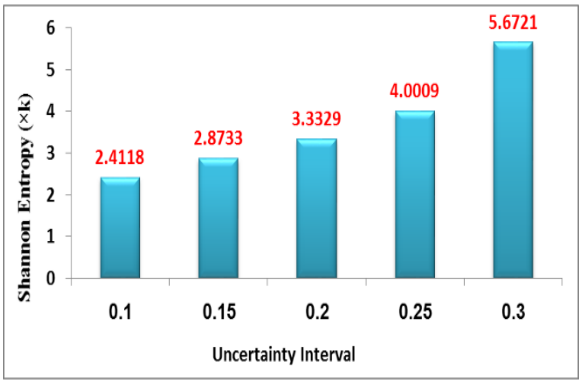

Fig. 4. Shannon-Entropy measure vs. uncertainty interval

5. Conclusion

The implication of fuzzy reliability analysis based on stability transformation method entails to the definition of uncertainty bounds for reliability index which yields more reasonable results compared to the classic methods of structural safety analysis methods. Moreover, sensitivity of fuzzy reliability index has been evaluated regarding to the different values of reliability interval. Results indicated that the epistemic uncertainty is more crucial in some engineering problems.