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Improving the Extended Finite Element Method in the Crack Problems via the Remeshing Process

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1-Introduction

The challenging and complex nature of the numerical analysis of crack problems has attracted the interest of many researchers in the past decades and several techniques have been proposed for these problems. One of these techniques is the extended finite element method in which the crack tip field modeling is improved by the enrichment of shape functions, and the crack can intersect the elements. On the other hand, we have adaptive the finite element method which aims to improve the accuracy of displacement and stress fields near the crack tip by remeshing the process. Researchers have reported the drawbacks of the two techniques. In this paper, the drawbacks of the previous techniques are covered with proper combination of these two techniques. By this combination, the crack can pass through the elements and there is no need for crack tracking by mesh. In addition, the estimated error is limited to desirable bands, and the stress intensity factor can be computed numerically with an acceptable accuracy.

2- Combined adaptive and extended finite element method

In the extended finite element method (XFEM), the displacement shape functions are enriched to the model of the discontinuities and cracks. Thus, there is no need to remesh the model in each step of the crack growth. These enrichment functions are categorized into two classes: the enrichment functions for elements that are intersected by crack and the function used for the element of the crack tip.

In the crack problems, the discretization error is high in the crack tip region due to the stress singularity in this region. Thus, the evaluation of the stress intensity factor is of difficulty and is less concerned with the theoretical values. Therefore, the adaptive finite element method will reduce the solution via the mesh modification and find the critical regions which demand finer mesh. In this paper, a recovery based method is employed to recover the finite element solution and achieve more accurate results. In this approach, the error is estimated as the difference between the recovered values and finite element solutions. After the error estimation, an optimum mesh is generated according to the estimated error. In this mesh, the elements with higher estimated error are refined and more course elements are generated in the regions with fewer errors.

The stress intensity factor is employed to illustrate the intensity of the singular stresses near the crack tip. The interaction integral is one of the most accurate techniques for evaluation of the stress intensity factor. This method is an improved version of J-integral method which evaluates the J-integral with an auxiliary field which is simpler and more accurate.

In this paper, the crack is modeled using XFEM and the crack passes through the elements. After commutating the stresses, the domain error is estimated. If the estimated error exceeds the error under discussion, the mesh is refined via adaptive mesh refinement and the problem is reanalyzed. Throughout these steps, the crack can pass through the elements. When the estimated error yields an acceptable value, the stress intensity factor will be calculated using the interaction integral method. With this combination, there is no need to track the crack path by elements and the stress intensity factor is evaluated on the base of more accurate stresses. In all of the analysis steps, the finite element mesh will remain unchanged and become independent from the crack path to benefit from XFEM. In case the mesh error exceeds the error in question, mesh will be regenerated. For example, it may demand only two steps of mesh refinement out of the ten steps of crack growth. The refined mesh does not conform to the crack path.

3- Numerical example

In this section, a numerical example is investigated so as to illustrate the accuracy and efficiency of the proposed technique. This example is one of the classic fracture mechanics problems and its theoretical stress intensity factor is available. It is a tensile rectangular plate with an edge crack. In this example, the adaptive and extended finite elements are combined and it is shown that in two steps of crack growth, the estimated error exceeds the error under consideration and the mesh refinement is needed. The refined meshes in these two steps are shown in Fig. 1. The dense mesh near the crack tip of each step indicates that the proposed algorithm has properly detected the critical points of the problem and reaches a proper mesh.

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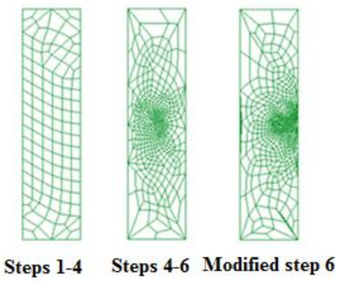

Fig.1 Mesh refinement steps

The variation of estimated error in different steps of the crack growth is shown in Fig. 2. It can be seen that in the steps where the error is beyond the error under consideration, the mesh refinement would reduce the error. The stress intensity factor is evaluated by the combined method and the results are given in Table 1 and are compared with theoretical values.

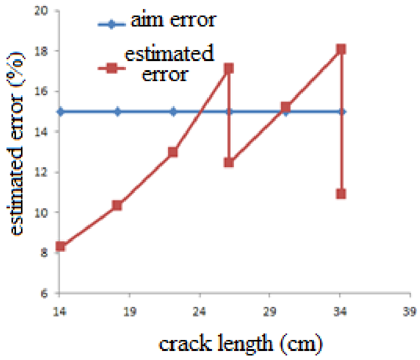

Fig.2 Variation of the estimated error with crack length

Table 1. Stress intensity factors in the edge crack problem

Crack growth step	Estimated error	K_I Numerical	K_I Analytical	K_{II} Numerical
1	7.47	7.55	8.69	0.365
2	10.51	9.73	11.77	-0.54
3	13.69	13.28	16.25	- 0.24
4	16.74	18.21	22.99	1.06
4 refined	10.02	20.36	22.99	0.52
5	12.95	30.73	33.238	1.68
6	17.55	32.58	48.60	6.11
6 refined	9.20	47.90	48.60	-1.6

4- Conclusions

According to the results obtained in this research, the following concluding remarks could be listed:

1. In the proposed method, the crack can be modeled, being independent of the structural mesh.
2. The fracture mechanics parameters were obtained more accurately with the proposed algorithm and the crack path is more coincident with the analytical results.
3. Adaptive mesh refinement is accomplished only in some steps where the estimated error exceeds the error under consideration.
4. The proposed algorithm automatically detects the critical regions of the problem and refines the mesh in such regions.