Improvement of Pareto Diagrams in Topology Optimization using Unstructured Polygonal Finite Elements

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1-Introduction

One of the approaches in weight reduction of structures is to introduce voids and gaps in the design domain. This basic idea has led to the formation of topology optimization algorithms. The main purpose of structural topology optimization is to find the optimized layout in a determined area to transfer the applied loads to the boundary areas and support locations. One of the problems frequently seen in topology optimization using common elements such as square or rectangular is the checkerboard phenomenon. In this uniform meshing, it may often happen that elements are connected to each other at least by one node and this leads to create some more restriction in flexibility of the final lay-out in the optimization process. Generally speaking, any discretization scheme that can better estimate the continuous design domain, would result in reducing the checkerboard phenomenon. A suitable solution to prevent such unwanted problems is the application of polygonal finite elements. Application of the polygonal elements without applying any filtering will solve the checkerboard phenomenon, the polygonal elements are either not connected to each other or are connected to each other by two nodes or a side.

Unstructured polygonal elements are more useful and practical in discretization of topology optimization through providing more flexible discretization of the complex domain. In order to form the irregular polygonal meshes, Voronoi diagram is generally used. The interesting feature of this method is that the random surfaces with geometric isotropy, are obtained by inserting the desired and fully randomly points. Subsequently Lloyd's algorithm is used to make the elements uniform. A polygonal mesh is created using the set of random points in the domain Ω, as well as auxiliary points for approximation of the boundary ∂Ω. Generally, the following method is suggested for initial meshing:

1. The interior of domain Ω, a set of random points, is created with the considered numbers. This point set is denoted by $P_{int.}$
2. To establish an appropriate approximation of the boundary of a domain, the interior points should be reflected about the edges of the domain. This auxiliary points set is denoted by $P_{aux.}$
3. The Voronoi diagram is constructed with point set $P= P_{aux} \cup P_{int}$.
4. A polygonal discretization of the domain is created by the cells associated with the points.

Voronoi diagram arrangement is determined completely by the generated points set. After that, using Lloyd's repetition designs for constructing polygonal meshes are obtained. The initial version of Lloyd's algorithm is as follows:

1. Construct the Voronoi diagram linked with the points
2. Compute the center of each cell
3. Replace the original point set by centroid points set (center of mass) and go to step 1 except convergence is reached.

In Lloyd's algorithm, the centroid of each part is computed as follows, and it is replaced by the initial points:

$$y = y_c \rightarrow y_c := \frac{\int_{V_y \cap \Omega} x\mu(x)dx}{\int_{V_y \cap \Omega} \mu(x)dx}$$

From Lloyd's algorithm that is shown in the following equation, it is seen that the energy function is decreased in consecutive iterations:

$$\varepsilon(P_{i+1}, \Delta) \leq \varepsilon(P_i, \Delta)$$

Fig. 1 shows the reduction of energy value function due to the generated point deviation in various repetitions.

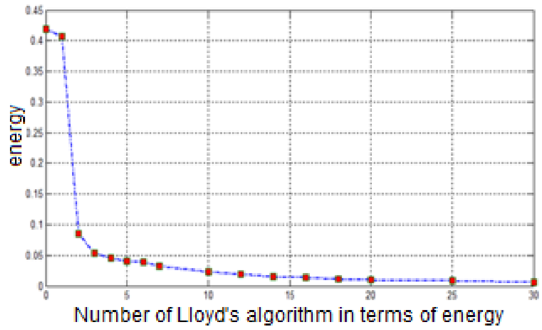

Fig. 1. Algorithm of energy reduction considering repetition of Lloyd's algorithm

It was seen that the main factor to generate the meshes is to use Voronoi diagram in order to discrete the domain. Lloyd's method is applied to create uniform seed distribution and subsequently to construct high quality mesh.

3- Multi objective topology

In the multi objective optimization problems, several objectives are optimized simultaneously. There are various methods to solve the multi objective problems. In this article, we used a method based on the concept of topological sensitivity (Suresh, 2010) for multi objective problems. The objective here is to develop a simple and efficient method to directly trace the Pareto-frontier for the two-objective topology optimization problems. Programs are developed in Matlab and a 450 line code was written for the generating unstructured polygonal mesh and Pareto-

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optimal tracing in multi objective topology for both cases of uniform and unstructured meshes. In MBB beam, the structure is optimized based on two objective functions; compliance and volume. The Pareto front is displayed in Fig. 2.

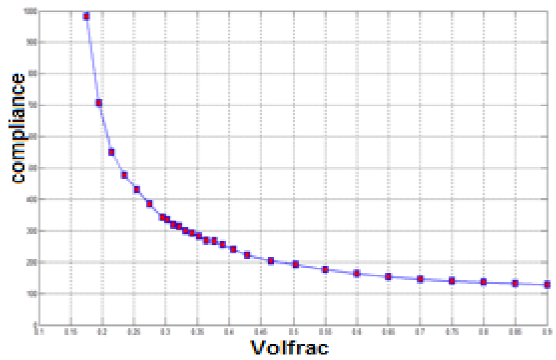

Fig. 2. Optimal tracing the MBB beam with unstructured polygonal elements

In order to compare and validate our results, the obtained Pareto front is compared with the Pareto front obtained from Suresh's method with square mesh and with the similar objective functions. Both examples have been performed for MBB beam with 1200 elements for polygon mesh and 20 x 60 numbers for square elements. Both Pareto graphs have been plotted in Fig. 3 for comparison.

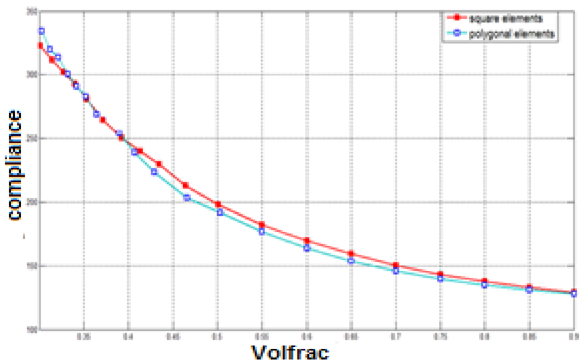

Fig. 3. Optimal comparison of Pareto MBB beam with unstructured polygonal elements and square elements.

As it can be seen in Fig. 3, the results of topology optimization with polygonal meshing is better that obtained from square meshing. The obtained Pareto graph from polygonal meshing method, corresponding to Pareto frontier is placed under Pareto graph obtained from the case of square meshing.

In order to present the efficiency of our proposed algorithms, run-time of MATLAB program are also measured and presented in Table 1. This table also displays the effects of number mesh elements for both cases of square and unstructured polygonal elements on the run time. As it can be seen, using unstructured polygonal elements results in the reduction of deployment time by more than one-third.

Table. 1. Comparison of the MATLAB run-time for unstructured polygonal and square elements

Number of elements	Unstructured polygonal elements	square elements
400	231 min	710 min
800	395 min	1231 min
1200	498 min	1497 min

4- Conclusion

To achieve the aim of decreasing structural weight, many studies have been done by designers. One of the proposed approaches in this case is to introduce gaps in the structures. Application of meshing approach with unstructured polygonal elements makes is possible to model and mesh every type of structure. Unstructured mesh is economical, because it results in a serious reduction of the computational time. Moreover, using the multi objective topology optimization with unstructured mesh, would give a better Pareto front and more accurate solutions, as compared to those using the mesh square.