

Coupled Analysis of Earth Dams and Estimating the Associated Pore Water Pressure Using Finite Element Method

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1- Introduction

A significant increase in pore water pressure during construction of earth dams may lead to the hydraulic fracture of dam body in pounding. Thus, having sufficient information about generation pattern of excess pore water pressure inside the core is essential. During the construction of earth dams, increased overheads caused by increased thickness of the embankment leads to the displacement of dam body and its foundation, as its increase rate is dependent on the permeability of materials. On the other hand, the high speed of the embankment construction is one of the factors that increase the pore water pressure inside the core. So, the interaction between fluid and soil skeleton should be considered in the analysis of dam construction. In the present study, dynamic and quasi-static form of Biot coupled equations were used to analyze the staged construction of earth dams by finite element method. So, construction of Daroongar dam was modeled in ten layers and obtained results of excess pore water pressure and displacements by coupled dynamic and quasi-static Biot equations were compared with instrumentation data. Finally, the actual amounts of horizontal and vertical permeability coefficients were determined for the materials by regression analysis.

2- Materials & Methods

For the analysis of saturated porous media, Biot proposed both equilibrium and continuity equations can be solved together. In his theory the governing equations of saturated porous media with a single fluid phase, generally water, are formulated as follows:

$$\sigma_{ij,j} + \rho \ddot{u}_i - \rho b_i = 0 \quad (1)$$

$$[K_{ij}(p_{,j} + \rho_f \ddot{u}_j - \rho_f b_j)]_{,i} + \alpha \dot{\varepsilon}_{ii} + \frac{\dot{p}}{c} = 0 \quad (2)$$

Where b_i is the body force per unit mass, K_{ij} is the dynamic permeability, ρ_f is the fluid density and ρ is the density of total composite, defined by

$$\rho \approx n\rho_f + (1-n)\rho_s \quad (3)$$

Where n denoting the porosity, ρ_s the density of solid particles and C is the combined compressibility of fluid and solid phases as it can be presented by

$$C \cong \frac{n}{k_f} + \frac{(1-n)}{k_s} \quad (4)$$

Eqs. (1) and (2) form the u-p formulation, which must be solved in a coupled manner. These equations involve two variables: u and p .

3-Numerical solution of governing equations

The finite element method has been used to solve the governing equations of (1) and (2). Spatial discretization of equations has been done by Galerkin method. In this study functions of first and second order are used for pressure and displacement, respectively. Discrete form of governing equations are derived as follows:

$$[M]\{\ddot{\bar{u}}\} + [k_m]\{\bar{u}\} - [Q]\{\bar{p}\} = \{f^{(1)}\} \quad (5)$$

$$[Q]^T\{\bar{u}\} + [k_c]\{\bar{p}\} + [S]\{\bar{p}\} = \{f^{(2)}\} \quad (6)$$

Where \bar{u} and \bar{p} are nodal displacement and pore pressure vectors, respectively. M , K_m , Q , S , and k_c are mass, stiffness, coupling, compressibility, and permeability matrixes, respectively. $f^{(1)}$ and $f^{(2)}$ are the nodal force vectors. They are expressed as follows:

$$[M] = \int (N^u)^T \rho N^u d\Omega \quad (7)$$

$$[k_m] = \int [B]^T [D] [B] d\Omega \quad (8)$$

$$[Q] = \int [B]^T [N^p] d\Omega \quad (9)$$

$$[k_c] = \int [B_p]^T [\kappa] [B_p] d\Omega \quad (10)$$

$$[S] = \int N^p \left(\frac{1}{k_f} + \frac{1-n}{k_s} \right) N^p d\Omega \quad (11)$$

$$f^{(1)} = \int (N^u)^T \rho b d\Omega \quad (12)$$

$$+ \int (N^u)^T \bar{t} d\Gamma$$

$$f^{(2)} = - \int (N^p)^T \nabla^T (k_s \rho_f b) d\Omega \quad (13)$$

$$+ \int (N^p)^T \bar{q} d\Gamma$$

In order to complete numerical solution, time integration of equations (5) and (6) are needed. In this study, Generalized Newmark time integration scheme has been used. By applying the GN22 method to soil displacement and GN11 to the pressure, parameters of acceleration, velocity, displacement, pore pressure change and the pore pressure at time t^{n+1} will be as follows;

$$\ddot{u}_{n+1} = \ddot{u}_n + \Delta \ddot{u}_n \quad (14)$$

$$\dot{u}_{n+1} = \dot{u}_n + \ddot{u}_n \Delta t + \beta_1 \Delta \ddot{u}_n \Delta t \quad (15)$$

$$\bar{u}_{n+1} = \bar{u}_n + \dot{u}_n \Delta t + \frac{1}{2} \ddot{u}_n \Delta t^2 \quad (16)$$

$$+ \frac{1}{2} \beta_2 \Delta \ddot{u}_n \Delta t^2$$

$$\dot{p}_{n+1} = \dot{p}_n + \Delta \dot{p}_n \quad (17)$$

$$p_{n+1} = p_n + \dot{p}_n \Delta t + \beta \Delta \dot{p}_n \Delta t \quad (18)$$

Parameters of β_1 , β_2 and β are in the range of 0 to 1. For unconditional stability it is required $\beta_1, \beta_2 \geq$

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0.5 and $\beta \geq 0.5$. By applying incremental method and assuming linear soil behavior, coupled equations in matrix form will be as follows:

$$\begin{bmatrix} [M]_{n+1} + \frac{1}{2}\beta_2\Delta t^2[k_m]_{n+1} & -\Delta t\theta[Q]_{n+1} \\ \beta_1\Delta t[Q]_{n+1}^T & S + \Delta t\theta[k_c]_{n+1} \end{bmatrix} \begin{Bmatrix} \{\Delta\ddot{u}_n\} \\ \{\Delta\dot{p}_n\} \end{Bmatrix} = \begin{Bmatrix} \{f^1\}_{n+1} - \{f^1\}_n + [Q]_{n+1}\{\dot{p}_n\}\Delta t - [k_m]_{n+1}(\{\ddot{u}_n\}\Delta t + \frac{1}{2}\{\ddot{u}_n\}\Delta t^2) \\ \{f^2\}_{n+1} - \{f^2\}_n - [H]\{\dot{p}_n\}\Delta t - [Q]_{n+1}\{\ddot{u}_n\}\Delta t \end{Bmatrix} \quad (19)$$

By eliminating expressions of acceleration (inertial forces) from the governing equations and ignoring the compressibility of the fluid and solid particles, quasi-static equations are obtained; which are often used in consolidation problems.

$$\begin{bmatrix} [k]_{n+1} & [Q]_{n+1} \\ [Q]_{n+1}^T & -\Delta t\theta[k_c]_{n+1} \end{bmatrix} \begin{Bmatrix} \{\Delta u\} \\ \{\Delta p\} \end{Bmatrix} = \begin{Bmatrix} \{f\}_{n+1} - \{f\}_n \\ \Delta t[k_c]\{p\}_t \end{Bmatrix} \quad (20)$$

4- Case study

Daroongar dam is an earth dam with a vertical clay core. Material properties of the dam body and its foundation are listed in Table 1. Due to the high length of the crest, calculations are done in plane strain condition for the highest dam cross section (cross section-a7, fig. 1). To simplify the calculations, three-dimensional effect of dam body is neglected. Construction of the dam is modeled in 10 layers in total duration of 28 months. For this purpose, model with fixed grid is considered, the weight of each layer assumed to be zero before construction and its actual weight is applied after construction. Dewatering of the reservoir is taken place 10 months after the end of construction.

Table 1. Material properties of Daroongar dam.

Material type	Core	Shell	Foundation
Drainage conditions	undrained	drained	drained
E(kPa)	22300	40000	17000
θ	0.25	0.2	0.3
k_x (m/s)	10e-9	10e-6	10e-9
k_y (m/s)	10e-9	10e-6	10e-9
ρ_s (kg/m ³)	2000	2100	1900
ρ_f (kg/m ³)	1000	1000	1000
K_f (Pa)	2.1e9	2.1e9	2.1e9
K_s (Pa)	1.0e20	1.0e20	1.0e20

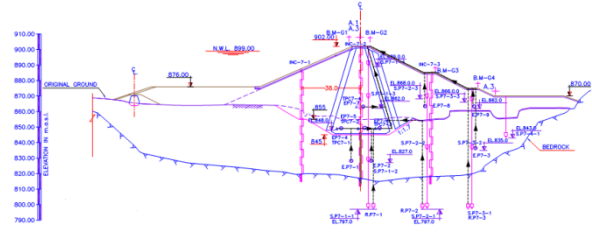


Fig. 1. The 7-a cross section of the Daroongar dam and its instrumentation

5- Conclusion

1. All diagrams of pore water pressure and displacements were ascending which is logical with respect to the increased embankment level and increased overhead.
2. At the higher levels, the results were more affected by the amount of horizontal permeability.
3. The final amount of horizontal and vertical permeability of the core materials were obtained by regression analysis as follows: $K_x = 2 \times 10^{-11}$ (m / sec), $K_y = 1 \times 10^{-11}$ (m/sec)
4. The amount of generated pore water pressure and its increase rate inside the core were depending on embankment level and distance from filters. So that at a certain level, the maximum amount of pore water pressure was in the middle part of the core and it was reduced at the periphery, this is completely consistent with reality.
5. On average, the results of dynamic analysis were obtained 20 percent more than the quasi-static one. At the end of construction, the maximum amount of generated pore water pressures in the quasi-static analysis differed 13 to 14 percent from instrumentation data, and in dynamic analysis, the results were 30 to 35 percent more than the instrumentation.
6. If the speed of construction of the embankment is high, dynamic equations must be used in analysis; and the results of quasi-static analysis will be acceptable only if the embankment speed is slow enough. Therefore, by considering the more number of layers and fairly long construction time, the construction of the dam can be analyzed by quasi-static equations.