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A New Formulation for Fictitious Mass of Viscous Dynamic Relaxation Method

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1-Introduction

Calculation of nodal displacements and member forces under applied loads are the aim of each structural analysis. Member forces could be written in terms of nodal displacements. Thus, nodal displacements are the main parameters, obtained from structural analysis. This goal is achieved by solving a simultaneous system of equations, obtained from the finite element or finite difference schemes, as follows:

$$SD = F = P \quad (1)$$

The main difficulty for solving Eq. (1) arises from nonlinear behavior. The explicit methods such as Dynamic Relaxation (DR) technique use vector operations to get answer of Eq.(2). This feature increases the efficiency and reduces the required memory so that the answer is obtained by simple calculations.

According to damping factor, Dynamic Relaxation method is categorized into two main branches, i.e. Kinetic Damping and Viscous Damping. Since the mechanical energy of a conservative system is constant, the potential energy of a position with maximum kinetic energy will be minimum which presents static equilibrium. This concept follows in the kinetic DR procedure.

In the viscous DR technique, by assuming fictitious damping for structure, the static system is transferred to an artificial dynamic space. Due to the existence of fictitious damping and absence of dynamic load, the transient response disappears after a while and finally, the steady- state response of the fictitious dynamic system that is the static equilibrium position of the main structure remains. For this purpose, the static system of Eq. (1) is shifted to a fictitious dynamic space, as follows:

$$M^n\ddot{D}^n + C^n\dot{D}^n + S^nD^n = F^n = P^n \quad (2)$$

In this paper, a new relationship is proposed for fictitious mass of Dynamic Relaxation (DR) method with viscous damping. For this purpose, a transformed Gershgörin theory is designed. Utilizing transformed Gershgörin theory, a new relationship is achieved for fictitious mass of viscous DR by formulating modified time step ratio. This procedure presents new algorithm for the viscous DR method.

2-Transformed Gershgörin Theory

It should be mentioned that the numerical stability of DR iterations depends on the upper bound of eigenvalues of fictitious dynamic system. In the DR technique, approximate methods such as Gershgörin theory are utilized for estimating the eigenvalues. This theory has an important weakness so that it may present negative or zero values for eigenvalues that is not possible for dynamic behavior of the system. This difficulty is removed by proposing a transformed Gershgörin theory. The transformed Gershgörin theory estimates the eigenvalues in three sub-domains;

$$S_{ii} > \sum_{\substack{j=1\\ j\neq i}}^{q} |S_{ij}| \qquad i = 1, 2, ..., q \quad (3)$$

$$\frac{1}{2}\sum_{\substack{j=1\\ j\neq i}}^{q} |S_{ij}| \le S_{ii} < \sum_{\substack{j=1\\ j\neq i}}^{q} |S_{ij}| \qquad i = 1, 2, ..., q \quad (4)$$

$$S_{ii} < \frac{1}{2}\sum_{\substack{j=1\\ j\neq i}}^{q} |S_{ij}|\ , \quad S_{ii} = \sum_{\substack{j=1\\ j\neq i}}^{q} |S_{ij}| \qquad i = 1, 2, ..., q \quad (5)$$

Utilizing the transformed Gershgörin theory and satisfying the stability conditions of DR procedure, a new time step ratio is proposed as follows:

$$\alpha = \underset{i}{\mathrm{Min}} \begin{cases} \dfrac{2 + 2\beta - \gamma_i^2 - 2(1+\beta)\sqrt{1-\gamma_i^2}}{\gamma_i^2} \\ -1 + 2\gamma_i^2 + 2\beta\gamma_i^2 - 2(1+\beta)\gamma_i\sqrt{1-\gamma_i^2} \\ \dfrac{8(1+\beta+\gamma_i+\beta\gamma_i) + \gamma_i^2 + 2\beta\gamma_i^2 - 4(1+\beta)\sqrt{\gamma_i^3 + 5\gamma_i^2 + 8\gamma_i + 4}}{\gamma_i^2} \end{cases} \quad (6)$$

3-The Proposed DR Algorithm and Numerical Results

Utilizing the modified time step ratio, the proposed algorithm for viscous DR method is as follows:

1. Assume initial values for artificial velocity (null vector), displacement (null vector or converged displacement on the previous increment, if available), fictitious time step and its ratio ($\alpha = \tau = 1$) and the convergence criterion for unbalanced force and kinetic energy ($e_R = 1.0E-6$) (and $e_K = 1.0E-12$).
2. For the first iteration, it is assumed that $\alpha = 1$, otherwise, calculate time step ratio from the proposed formulation.
3. Construct tangent stiffness matrix and internal force vector.

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4. Apply boundary conditions.
5. Calculate the out-of-balanced force vector.
6. If $\sqrt{\sum_{i=1}^{q}(r_i^n)^2} \leq e_R$, go to (14), otherwise, continue.
7. Construct the fictitious diagonal mass matrix.
8. Form artificial damping matrix.
9. Update artificial velocity vector.
10. $\tau^{n+1} = \alpha\tau^n$
11. If $\sum_{i=1}^{q}(\dot{D}_i^{n+1/2})^2 \leq e_K$, go to (14), otherwise, continue.
12. Update displacement vector.
13. Set $n = n + 1$
14. Print the result of the current increment.
15. If increments are not complete, go to (1), otherwise, stop.

To evaluate the numerical efficiency of the proposed method, some 2D and 3D truss and frame structures are analyzed with elastic linear and geometrically nonlinear behavior. For this purpose, a computer program is written based on the proposed DR algorithm, using the Fortran Power Station software. The prepared program utilizes the finite element method for modeling the structures.

The convergence of the prepared DR program is controlled by calculating kinetic energy and residual force of fictitious dynamic system at the end of each iteration.

Numerical studies prove that the proposed method is completely stable so that convergence to the static equilibrium position could be always achieved. In other words, the proposed DR algorithm guarantees the DR stability. Moreover, the convergence rate of the suggested DR process could be evaluated by comparing the number of convergence iterations. The results show that by using the proposed algorithm for fictitious mass, the convergence rate of the viscous DR method is improved so that the proposed algorithm presents the structural response with lower iterations in comparison with other common DR techniques.

4-Conclusions

By using transformed Gershgörin circles theory, which was previously used in the kinetic DR technique, a new formulation was proposed here for the fictitious mass of viscous Dynamic Relaxation method. Accordingly, a new algorithm that uses modified time step ratio was introduced for viscous DR technique. For numerical verification of the proposed algorithm, some structures such as trusses and frames with linear and nonlinear geometrical behavior were analyzed. The results could be listed as follows:

1. By performing the proposed formulation, new fictitious time step was achieved for Dynamic Relaxation method.
2. The proposed viscous DR algorithm is completely stable so that convergence to the steady state is achieved in all numerical examples.
3. The convergence rate of the proposed method is more than the well-known viscous DR techniques. In this case, a considerable reduction has occurred in required convergence iterations. In other words, the proposed technique has suitable efficiency and higher convergence rate in comparison with other common viscous Dynamic Relaxation method.
4. The proposed DR method does not impose any additional calculations to the common Dynamic Relaxation algorithm. As a result, the analysis time of the new scheme is less than the well-known DR methods.