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Evaluating Failure Probability of Rectangular Plates with Different Boundary Conditions using the Dynamic Finite-step Length Method

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1-Introduction

Rectangular plates are widely used to construct various structural/mechanical components in engineering systems. The box girder of log-span beiges, composite decks of buildings, plate aircraft panels, and steel shear walls to provide the lateral resistance of stories can be made based on plate elements. Plate structures involve various uncertainties to evaluate their performance including their dimensions, loads, analytical model and boundary conditions. Usually probabilistic models are applied to consider different uncertainties of plate elements under serviceability limit state surface. Design optimization methods with uncertainties have been used to evaluate reliable performance of probabilistic model of plates.

The structural reliability analysis-based probabilistic model can be used based on the performance function to evaluate the safety levels of plates for considering different uncertainties. Generally, the analytical methods (such as: the first order reliability method, the first order second moment, the advanced mean value, and the second order reliability method) and simulation approaches (for example: Monte Carlo simulation, weighted simulation, subset simulation, and importance sampling) can be implemented to evaluate the failure probabilities of the structural /mechanical engineering problems. The first order reliability method (FORM) is widely used for structural reliability analysis due to its simplicity and efficiency by the following approximated integration for estimating the failure probability:

$$P_f = P[g(X) \le 0] = \int_{g(X)\le 0} f_X(x)dX \approx \Phi(-\beta) = \Phi\left(-\left\|U^*\right\|\right) \quad (1)$$

Commonly, the iterative FORM formula-based HL-RF method is applied for structural reliability analysis. However, the HL-RF may produce unstable results as chaotic and periodic solutions for highly nonlinear performance function. In the improved HL-RF, the convergence of the HL-RF method can be monitored using merit function. The stability transformation method, finite-step length, modified chaos control, conjugate chaos control, conjugate stability transformation methods are the modified versions of FORM to improve its robustness and efficiency for highly nonlinear reliability problems. The conjugate search direction has been formulated, more computationally complex formulations. The steepest descent search direction for formulation of FORM are simpler than the conjugate reliability methods. The simplicity, robustness and efficiency of structural reliability methods are the major issues for selecting an iterative FORM formula to evaluate the failure probability of complex engineering systems.

Rectangular plates with different boundary conditions under the displacement performance function can be used for reliability analysis of real engineering problems. In this paper, a simple limit state function is defined to approximate the maximum displacement of plates subjected to distributed load based on the finite element model. The finite-step length approach is modified using an adaptive step size with Armijo line search. The results indicate that the boundary conditions and the arrangement of the plate can be used to improve the reliability levels of plates.

2- Dynamical finite-step length method

The dynamical finite-step length method (DFSL) is given by the following iterative formula of FORM:

$$\boldsymbol{U}_{k+1} = \frac{g(\boldsymbol{U}_k) - \nabla^T g(\boldsymbol{U}_k)\boldsymbol{U}_k}{\nabla^T g(\boldsymbol{U}_k)\boldsymbol{\alpha}_{k+1}^{\lambda}}\boldsymbol{\alpha}_{k+1}^{\lambda} \quad (2)$$

where $\boldsymbol{U}_{k+1}$ is the new point in the normal standard space, $\boldsymbol{\alpha}_{k+1}^{\lambda}$ stands the normalized finite-search direction vector, which is computed as follows:

$$\boldsymbol{\alpha}_{k+1}^{\lambda} = \frac{\boldsymbol{U}_{k+1}^{\lambda}}{\left\|\boldsymbol{U}_{k+1}^{\lambda}\right\|} \quad (3)$$

where $\boldsymbol{U}^{\lambda}$ the finite-point, which is given as:

$$\boldsymbol{U}_{k+1}^{\lambda} = \boldsymbol{U}_k - \lambda_k \nabla g(\boldsymbol{U}_k) \quad (4)$$

In which, λ_k is the dynamical finite-step length. The $\boldsymbol{U}^{\lambda}$ is located on the previous point when $\lambda_k = 0$. This means that the new point is a fixed point. A suitable value of λ_k can be used to control the convergence properties of the proposed DFSL method. In this paper, the Armijo rule is used to select the appropriate step size as follows:

$$\lambda_k \ge \frac{g(\boldsymbol{U}_k + \lambda_k \boldsymbol{d}_k) - g(\boldsymbol{U}_k)}{m\left\|\nabla g(\boldsymbol{U}_k)\right\|^2} \quad (5)$$

Where $\boldsymbol{d}_k$ is the search direction vector as $\boldsymbol{d}_k = \boldsymbol{U}_k - \boldsymbol{U}_{k-1}$, m is a real positive value and $\nabla g(\boldsymbol{U}_k)$ is the gradient vector of the limit state function at point $\boldsymbol{U}_k$. Therefore, Eq. (6) can be simplified as follows:

$$\lambda_{\max} \ge \frac{M}{\left\|\nabla g(\boldsymbol{U}_{\boldsymbol{U}=\mu})\right\|^2} \quad (6)$$

where M is a positive large value, i.e. 1-100 and the value of 15 is used for it in the current study and

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$\nabla g(\boldsymbol{U}_{U=\mu})$ is the gradient vector of the limit state function at the mean point. Therefore, the initial finite –step length is dynamically computed as follows:

$$\lambda_0 = \frac{15}{\left\|\nabla g(\boldsymbol{U}_{U=\mu})\right\|^2} \tag{7}$$

The dynamical step size in Eq. (7) should satisfy the sufficient descent condition to provide global convergence of the DFSL formula as follows:

$$\lambda_k = \begin{cases} \lambda_{k-1} & if \quad \left\|\boldsymbol{U}_{k+1} - \boldsymbol{U}_k\right\| < \left\|\boldsymbol{U}_k - \boldsymbol{U}_{k-1}\right\| \\ 0.75\lambda_{k-1} & \text{otherwise} \end{cases} \tag{8}$$

The DFSL method is as simple as the HL-RF method and is simpler than the conjugate reliability methods. The step size using Eq. (8) can be tended to zero when $k \rightarrow 0$. Thus, this iterative method can be provided a fixed point.

3- Performance function of the rectangular plate

The limit state function of a plate under distributed load q and the boundary conditions 1-2-3-4 for displacement performance function which is schematically shown in Fig. 1 is defined as follows:

$$g = \frac{a}{R} - \alpha \frac{qa^4}{D} \tag{9}$$

where D is the stiffness of the plate which is given as $D = Et^3/12(1-\nu^2)$, ν is Poisson's ratio, E is module of elasticity, t is plate thickness and R is the limited condition of the plate displacement that is selected based on the code design to be 240 and 360. α is the analytical coefficient which is computed using the finite element model of the plate with different conditions. The limit state function in Eq. (10) involves seven basic random variables including E, t, ν, α, a, q, and R with normal and log-normal random variables.

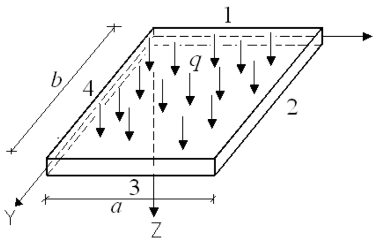

Fig. 1 Schematic view of rectangular plate with boundary 1-2-3-4

4- Reliability results

Figure 2 illustrates the failure probability of squared plates with different boundary conditions for three control coefficients as R= 240, R=360 and R=440. As seen, the failure probabilities for palates with the free (F) boundary conditions are more than that of other plates with simple supported (S) and clamped (C) boundary conditions. The plate with clamped conditions (CCCC) has a failure probability form order -8 for R=360 while the failure probability is increased to order -5 for plates with boundary CCFC. Consequently, the boundary conditions of the plate can be controlled to reliable levels as well as the resistances and dimensions of the plates. The type and arrangement of boundary conditions for controlling the maximum displacement of plates can be used to remarkably improve the reliability levels of the plate. The plate with the SCSC conditions can be used to improve the failure probability about 20% compared to the one with the SSCC conditions. It can be used to improve the reliability levels of the plate with free conditions with respect to a suitable arrangement of other conditions with clamped (C) boundary e.g. CSCF which can provide us with a failure probability of order -5.

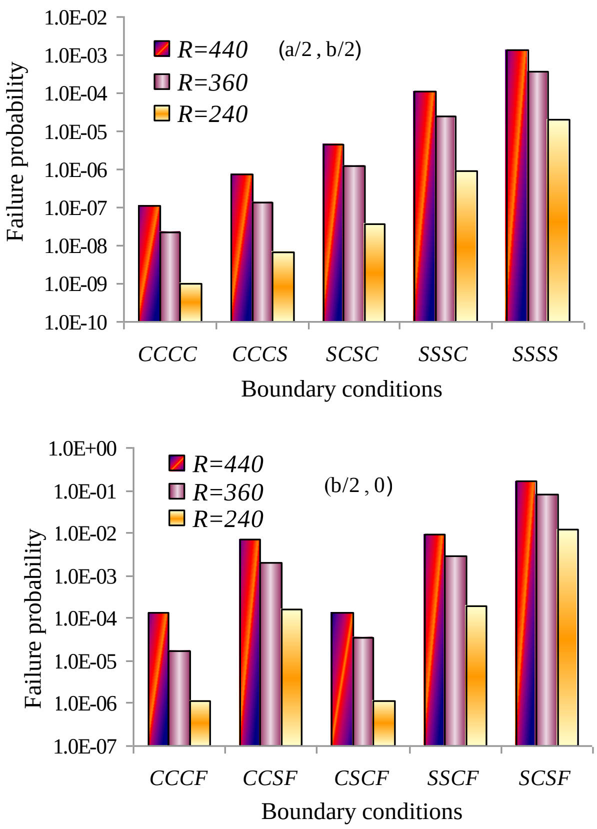

Fig. 2 Failure probabilities of the plate with respect to different boundary conditions

5- Conclusions

The failure probabilities of rectangular plates were evaluated using an iterative FORM formula-based dynamic finite- step length (DFSL) method. The DFSL is formulated based on dynamic step size, which is formulated using the Armijo rule and sufficient descent condition. The failure probability is more sensitive to dimensions than the distributed load compared to other conditions. The free boundary conditions strongly increase the failure probabilities. The suitable reliable levels are obtained using the boundary conditions CSCS, SSCS, CCCC, and CCCS. The locations of clamed boundary with another condition can be used to remarkably improve the safety levels of plates e.g. CSCF.