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Determination of Critical Slip Surface in earth Slopes using Alternating Variable Local Gradient

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1- Introduction

Stability analysis of soil slopes is one of the major issues in the science of soil mechanics. The limit method is widely used in slope's stability analysis. This method is divided into the limit equilibrium and the limit analysis methods. The limit equilibrium method is widely used for slope stability analysis to obtain the factor of safety. In slope stability analysis the minimum factor of safety that is critical slip surface is obtained. The shape of critical slip surface is usually considered to be a circular shape for simplicity. The concept of optimization-based methods have been used extensively in the analysis of earth slopes for determining the critical slip surface due to their higher efficiency and accuracy compared with other methods. The slope stability analysis is used for determining the critical slip surface using an optimization method which has been addressed as new work in this paper. The shape of the slip surface in this paper is line-segments shape. In this method, at first the circular critical slip surface is determined. Then, the appropriate number of nodes on the critical circular slip surface is selected and by connecting them to each other, line-segments slip surface is obtained. The line-segments slip surface is optimized using the Alternating Variable Local Gradient (AVLG) method to obtain critical line-segments slip surface. The DOSS program is written by the authors in FORTRAN to obtain the critical line-segments slip surface. This program may be used for non-homogeneous, saturated and unsaturated soils.

2- Determination of Critical Line-Segments Slip Surface

The methods based on optimization methods have more efficiency and accuracy compared with other methods to determine the critical slip surface. This optimization is difficult and complex. One of the problems in optimization of slip surface is local minimums. Local minimums are places where the factor of safety is not minimum.

In this paper, the Alternating Variable Local Gradient method is used to obtain the critical slip surface in earth slope. The Alternating Variable Local Gradient method is a non-linear programming approach in optimization. This method can simply escape from many local minimums.

3- Local Gradient Method and 2D-AVLG Method

The local gradient method is based on the Univariate method. In this method, one variable is moved in order to be optimized while the other variables remain fixed. Then, another variable is selected for optimization while the other variables again remain fixed. This process continues until all the variables are optimized by the end of the first cycle. Then, the optimization process of the second cycle is initiated. This process is also iterated until the movement of variables in the new cycle has no effect on the optimization of the objective function (safety factor).

The Univariate method is summarized as follows:

1. Choose an arbitrary starting point $\mathbf{X}_i$ and set $i = 1$
2. Find the search direction $\boldsymbol{S}_i$

$$\mathbf{S}_i^T = \begin{cases} (1,0,0,\ldots,0) & \text{for} \quad i=1,n+1,2n+1,\ldots \\ (0,1,0,\ldots,0) & \text{for} \quad i=2,n+2,2n+2,\ldots \\ (0,0,1,\ldots,0) & \text{for} \quad i=3,n+3,2n+3,\ldots \\ \vdots & \\ (0,0,0,\ldots,1) & \text{for} \quad i=n,2n,3n,\ \ldots \end{cases} \tag{1}$$

3. Find the optimal step length λ_i^* such that

$$f(X_i \pm \lambda_i^* S_i) = \min(X_i \pm \lambda_i S_i) \tag{2}$$

where + or - sign has to be used depending upon whether $+\mathbf{S}_i$ or $-\mathbf{S}_i$ is the direction for decreasing the function value.

4. Set $\mathbf{X}_{i+1} = \mathbf{X}_i \pm \lambda_i^* \mathbf{S}_i$, depending on the direction for decreasing the function value, and $f_{i+1}=f(\mathbf{X}_{i+1})$
5. Set the new value of $i = i + 1$ and repeat from step 2.

Continue this procedure until no significant change is achieved in the value of the objective function.

In summary, the AVLG method for finding the most critical line segments slip surface can be expressed as follows:

1. Set i=1
2. Finding the circular critical slip surface by using the Grid Search method, or any other method, and taking it as the initial slip surface.
3. Selecting proper nodes on the circular critical slip surface and connecting them to each other (the number of the selected nodes plays a significant role in the optimization process. It is recommended to select more nodes on the weak layers in non-homogeneous soils). $\mathbf{Z}_i$ denotes the coordinates of the initial selected nodes.

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$$\mathbf{Z}_i = (x_1, y_1, x_2, y_2, \ldots, x_n, y_n) \quad (3)$$

4. Finding the best location for the first node on the slope boundary.

The new coordinates of the slip surface are as follows:

$$\mathbf{Z}_i^* = (x_1^*, y_1^*, x_2, y_2, \ldots, x_n, y_n) \quad (4)$$

5. Finding the best location for the next node of the slip surface while also keeping the other nodes fixed results in a lower factor of safety. The best location for each internal node is obtained by its moving in the negative direction of the local gradient vector. The relation for the negative direction of the local gradient vector is as follows:

$$\mathbf{S}_k = -\mathbf{G}_k = -\left\{\frac{\partial Fs}{\partial x_k}, \frac{\partial Fs}{\partial y_k}\right\}^T \quad (5)$$

For example, node 2 moves from its initial coordinates (x_2, y_2), to its new coordinates (x^*_2, y^*_2), where it yields a lower safety factor. Thus, the new coordinates of the slip surface are as follows:

$$\mathbf{Z}_i^* = (x_1^*, y_1^*, x_2^*, y_2^*, x_3, y_3, \ldots, x_n, y_n) \quad (6)$$

6. Finding the best location for the subsequent internal node while other nodes remain fixed. This process is iterated for the rest of the internal nodes. The new coordinates of the slip surface are as follows:

$$\mathbf{Z}_i^* = (x_1^*, y_1^*, x_2^*, y_2^*, \ldots, x_k^*, y_k^*, \ldots, x_n, y_n) \quad (7)$$

7. Find the best location for the last node on the slope boundary. In this step the first optimization cycle is terminated. The new coordinates of the slip surface are as follows:

$$\mathbf{Z}_{i+1}^* = (x_1^*, y_1^*, x_2^*, y_2^*, \ldots, x_{n-1}^*, y_{n-1}^*, x_n^*, y_n^*) \quad (8)$$

8. Set i=i+1
9. Steps 4 to 7 are repeated for several cycles until the difference between the safety factors of the last two cycles is less than $\varepsilon=1\times10^{-5}$. Or

$$|FS(\mathbf{Z}_{i+1}^*) - FS(\mathbf{Z}_i^*)| < \varepsilon \quad (9)$$

FS ($\mathbf{Z}^*_{i+1}$) = the factor of safety for the last optimization cycle,

FS ($\mathbf{Z}^*_i$) = the factor of safety for the penultimate optimization cycle.

The slip surface associated with the last factor of safety is taken as the most critical slip surface.

5- Conclusion

The Alternating Variable Local Gradient method is able to obtain the minimum factor of safety and critical line-segment slip surface. The factor of safety is lower r than those obtained using critical circular slip surface. It should be noted that the line-segments slip surface is more consistent with the actual slip surface in nature and it is more reliable and flexible than the others. DOSS program is written by the authors whose results are in good agreement with the results obtained from other methods.