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Efficiency Improvement of the Structural System Identification by Reducing Singularity of the Response Matrixes in Inverse Solution of Equations of Motion

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1- Introduction

Structural system identification is known as a prelude of damage detection and structural health monitoring in which identification is operated on the base of inputs and outputs data, and occasionally is needed to find the inverse solution of system transfer function. Generally in the system identification process, the inverse problem is often an ill conditioned problem from a mathematical point of view. This paper presents a method for identification of linear system physical parameters (structural mass, damping and stiffness matrices) using the inverse solution of the equation of motion in the frequency domain, by focusing on the reduction of the ill conditioning effect. The method utilizes the measured responses from the forced vibration test of the structure in order to identify system properties and detect the probable damages.

Input and output data is gathered in an augmented matrix [A|b] and the large amount of this data causes the ill conditioned problem. Moreover, as an inevitable problem, there is a noise in the measurement that causes some discrepancies in the results of identification. Ill conditioning causes instability in the results of identification, the instability and noisy result reduces the validity of the results and accordingly it will be a worthless statistical method in system identification (SI). An algorithm is presented in this paper to improve the ill conditioning problem that is a special upper triangularization matrix method. The proposed algorithm can identify parallel and pseudo parallel vectors in the coefficient matrix of linear equations. By removing these linearly dependent vectors and thus reducing the degree of singularity of the matrix, stabilization results which is a key objective in numerical linear algebra. In order to perform an optimal estimation of identification results, least-squares and penalty function methods are used. The validity and efficiency of the reduced singularity of the matrix method is tested on a non-shear eight frame structure by using the direct model updating method. The aforementioned structures have a non-proportional damped matrix and are subjected to sweep harmonic forces. The results show that the proposed algorithm improves the stability of the estimation and the answer is quite useful.

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2- Direct System Identification Method

The direct identification method can be used to estimate full system properties including mass, M, damping, C, and stiffness, K, matrices of the system, based on the inverse solution of the equation of motion with the following formulations will be used.

The equations of motion for a viscously damped linear system with "n" number of degrees-of-freedom and "m" time step of input and response can be written as:

$$M_{n\times n}\ddot{X}_{n\times m} + C_{n\times n}\dot{X}_{n\times m} + K_{n\times n}X_{n\times m} = F_{n\times m} \quad (1)$$

By using the properties of blocked matrices, and defining $R_{m\times 3n} = [\ddot{X}^T_{m\times n} \quad \dot{X}^T_{m\times n} \quad X^T_{m\times n}]$ and $Q_{3n\times n} = [M_{n\times n} \quad C_{n\times n} \quad K_{n\times n}]^T$, Equation (1) yields:

$$R_{m\times 3n}Q_{3n\times n} = F^T_{m\times n} \quad (2)$$

R is generally an ill-conditioned matrix and a small change in R can cause a large change in $R^{-1.}$ Therefore, the accuracy and precision of Equation (2) is very sensitive to noise in the R. The residual force in the equation of motion in the frequency domain and as regards Ghafori Ashtiany and Ghasemi [1] showed that to obtain the best estimation for the linear system properties, minimization of the real or imaginary part of residual forces approximately lead to identical results of the physical parameters. Thus, it can be expressed as follows:

$$E_{R_{m\times n}} = R_{R_{m\times 3n}}Q_{3n\times n} - F^T_{R_{m\times n}} \quad (3)$$

The optimization objective function after minimization of the residual force and with regard to the condition of the unknown matrix (symmetrical system property matrices and diagonal mass matrix), is constrained by optimization as a penalty function and it is expressed as follows:

$$f = \sum_{k=1}^{m}\sum_{j=1}^{n} E_{kj}^2 + R_p\left(\sum_{i\neq j=1}^{n} Q_{ij}^2 + \sum_{i=n+1}^{2n}\left(Q_{ij} - Q_{j-n,i+n}\right)^2 + \sum_{i=2n+1}^{3n}\left(Q_{ij} - Q_{j-2n,i+2n}\right)^2\right) \quad (4)$$

where R_p is the coefficient of the penalty function and it refers to the value of the objective function. Consequently, the Optimum solution, is available with a process of differentiation ($\frac{\partial f}{\partial Q_{ij}} = 0$) and it can be written as follows:

$$\frac{\partial f^2}{\partial Q_{ij}} = 0 \Rightarrow pR_RQ = pF_R^T \quad (5)$$

where p is a preconditioner such as the penalty function.

3- Reduced Singularity of the Matrix Algorithm

The main idea of this paper is presented based on the reduced singularity of the matrix RT, which uses principles of linear algebra. In the proposed method, we identify numerical dependent column vectors in the matrix R^T, and remove them from it. In fact, in the process of optimization of the problem (Equation 2) each of the column vectors, is an equation that is embedded in the rows of the matrix R. In the process of solving the resulting equation, the number of equations is too many and we aim to reduce the number of equations in a similar fashion by utilizing linear algebraic techniques. This has two advantages, first, it can make less rotation by the affected preconditioner in equation (5) in the coefficient matrix. Therefore there will be less errors in the resulting unknowns problem (Q). This arises from the idea that parallel or Pseudo parallel vectors in the coefficient matrix have been removed. Second, with reducing the degree of ill-conditioning of the problem, stability of the results will be enhanced.

ALGORITHM: *let A be an n×n (square) matrix, and matrix B is the result of upper triangularization by following method on the matrix A, then i'th row of A ($1 \le i \le n$) is linear dependent if i'th column to end'th column of i'th row of B are zero.*

Method of upper triangularization:

1. *from The three elementary row operations ($1. e_{(p,q)}$, $2. e_{\lambda(p)}$, $3. e_{(p)+\lambda(q)}$), The second and third are used (dont interchange of two rows of A).*
2. *Be used $A_{i,i}$ as a pivot element and befor pivoting, if possible the $A_{i,i}$ to be non-zero ($1 \le i \le n$).*
3. *In the process, if the $A_{i,i}$ was zero, be used $A_{i+1,i+1}$ as a pivot element.*

In order to use this algorithm to solve the equation (5), we must be careful about two things. First, the matrix R must be an $m \times n$ $(m >> n)$, thus initially it should be partitioned into several $n \times n$ sub matrices. Second, in the case of $\lambda_1 A_1 + \lambda_2 A_2 + \cdots + \lambda_t A_t + A_i = 0$ the matrix A is rank deficient because one or more rows and columns of A are linear combinations of some or all of the remaining rows and columns. Here, the matrix A having a cluster of small singular values and there is well-determined gap between large and small singular values of A and it causes one type of system of equations with an ill-conditioned coefficient matrix (A). Another type of ill-conditioning of matrix A exists in ill-posed problems. In this case, rank and numerical rank are full, but the ratio between the largest and the smallest singular values is large and we have numerical instability in the solution of the system of equations. This implies that one or more rows and columns of A are numerically dependent on other rows and columns. Thus, they can be written as $\lambda_1 A_1 + \lambda_2 A_2 + \cdots + \lambda_t A_t + A_i = \varepsilon$. In the sub matrices of R, we encounter with this situation and finding a suitable estimate for ε is very important.

4- Numerical Example

The applicability of the proposed method on the identification of a general 8-story non-shear planar frame with non-proportional damping matrix as shown in Figure 1 is presented. Considering the rigid diaphragm effect, horizontal displacements are constrained at the story levels. To constrain the number of DOF's of the model to eight, the system masses are lumped at the story levels. The exact stiffness matrix of the model is defined using the FE model. Harmonic sweep (whose frequency sweeps from 1 to 10 Hz) input forces are applied at the first story level and the corresponding acceleration responses were obtained. Considering that the measured responses are noisy, Gaussian random white noise has been added to the calculated responses. The level of the noise is defined as the root mean square (RMS) of noise with respect to the RMS of the structure response and input forces. In each case, the amount of error of the system properties is calculated based on the mean error of the diagonal elements of property matrices. In each case, the amount of error of the system properties is calculated based on the mean error of the diagonal elements of property matrices. The method allows the complete identification of the structural mass, damping and stiffness matrices by direct solution of differential equations of motion. The structural system identification sensitivity is presented in the Figure shown below in two cases using and not using the reduced matrix singularity method when data is disturbed with noise (0~10%), regularization and the reduced error in the results is observed.

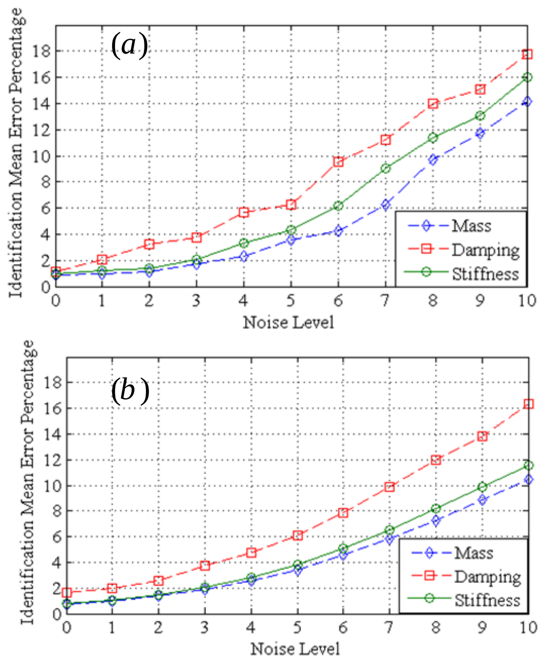

Fig. 1. Mean error percentage in system physical parameters identification for the case of 1~10% noises level: (a) not using reduce matrix singularity, (b) using reduce matrix singularity