continuous space and the new position of the particles in continuous space is obtained. Since the problem of finding the best generators need to search in discrete binary space, it is required to improve CSS for this problem. In the proposed model, the decision variables are defined as binary parameters shown by g_{nk} . The algorithm starts with an initial population of particles whose positions are determined randomly in the feasible space. For each null basis, the members which satisfy all of the below mentioned conditions can be placed on the list of feasible generators.

1- To insure the independency of columns, the members which have already been selected as generators of the previous null bases cannot be selected anymore.
2- For each null basis, one generator must be selected.
3- The feasible members as a candidate for each null basis generator are influenced by the earlier SESs generators.

4. Numerical Results

To illustrate the performance of the proposed algorithm and to provide a measure of its efficiency, some numerical examples are solved based on the proposed algorithm.

In order to present the elapsed run time of the algorithm, a unit block consisting of 4 nodes and 6 members is considered and sample structures are composed of equal number of such a unit in x and y directions, as shown in Fig. 1.

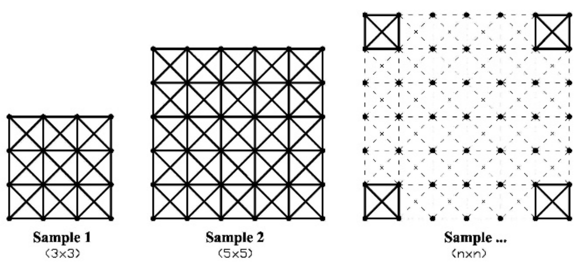

Fig. 1. The geometry of sample structures

The obtained results are shown in Table 1. As it is shown, the null bases matrix is approximately 95% sparse for the structure with 100 nodes.

Table 1. Optimal solutions based on the proposed algorithm

Node Numbers	Matrix Dimensions	Number of Samples	The Proposed Algorithm	
			Optimal Solution (Sparsity Percentage)	Average of the Elapsed Run Time (sec)
16	42×13	10	84.27	46.16
36	110×41	10	92.08	211.59
64	210×85	10	94.40	1097.47
100	342×145	10	95.31	3608.72

5. Conclusion

In this paper, a heuristic algorithm is presented to find the sparse null bases vectors. The main features of the presented approach are as follows:

1- The sparse null basis matrix leads to optimal flexibility matrix. Therefore, it makes an efficient force method feasible for the structural analysis.
2- To design constructive algorithm, the independency condition is satisfied based on basic rules of algebra. Each vector is normalized based on an entry which is called generator and then the orthogonality condition is satisfied.
3- The generators sequence is chosen as decision variable in the optimization model, and then the optimization problem is solved by using the modified CSS algorithm.
4- Since the system of linear equations consists of equilibrium conditions and uniqueness of generators is an underdetermined system, a linear mixed integer programming model is developed to find the solution with a large number of non-zero entries.
5- The numerical examples show the efficiency of the proposed algorithm to find a sub-optimal solution even for the truss with opening in its geometry.

Formation of Sparse Null Basis Matrix for the Efficient Force Method Using Charged System Search Algorithm

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1. Introduction

The force method was used extensively until 1960. After this, the advent of the digital computer and the amenability of the displacement method for computation attracted the attention of most researchers. As a result, the force method and some of the advantages it offers in non-linear analysis and optimization have been neglected. Although due to these changes, the progress of the force method was slow, lots of researches have attempted to improve this method to be suitable for the computer programming.

The first algebraic method was proposed by Denke. In general, algebraic methods are simple, but these methods need a large storage requirements and a higher number of operations when compared to others.

The main problem in the application of the force method is the formation of a self-stress matrix corresponding to a sparse flexibility matrix. Therefore, different approaches are adopted for the force method of structural analysis.

In this paper, a new approach is proposed for the formation of sparse null basis matrix corresponding to the efficient force method. The problem of optimal flexibility matrix formation is defined as a linear mixed integer programming model in which, the objective function is to minimize the number of nonzero entries in the null basis matrix, while the independency and equilibrium conditions are considered as constraints in the optimization model.

Although the problem of finding sparse solution of a flexibility matrix belongs to the class of NP-hard problems, the sub optimal solution can be obtained accurately by using the proper Meta heuristic algorithm.

In this paper, Charged System Search (CSS) algorithm is used for solving the proposed optimization model. In recent years, this algorithm has been used widely in different structural problems. These studies show that the CSS algorithm outperforms its rivals. The CSS algorithm is initially developed for continuous search space, but in this paper it is changed to be used for binary discrete variable.

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The presented method is employed in some pin-jointed truss structures with internal static indeterminacy.

2. Formulation of the Null Basis Matrix via an Optimization Model

Consider a structure S with M members and N nodes, which are $\gamma(S)$ times statically indeterminate. The overall flexibility matrix of the structure (G) is defined as $G = B_1^t F_m B_1$. B_1 .is called a self-stress matrix as well as the null basis matrix. Each column of B_1 is known as a null vector. B_1 is a rectangular matrix having n rows and $\gamma(S)$ columns.

For the efficient force method, the G matrix shall be generated sparse and well-conditioned. For the sparsity of G, one can search for a sparse B_1 matrix, since the F_m matrix is fixed for a specific structure.

Therefore, the optimization problem is defined to find the sparse null vectors, while the independency and equilibrium conditions are satisfied. For a null vector, no applied load is required, thus the equilibrium conditions can be expressed as $AB_1 = 0$

A is an $n \times m$ matrix with rank of n. There are $m - n = t$ independent columns of B_1 which will satisfy this equation, thus forming a set of null vectors.

Let us denote the columns of matrix B_1 by S_i. To satisfy the independency of B_1 columns, one particular element is selected in each column as “Generator”, the generator entry is 1 in its corresponding column while that is equal to zero in the other columns. After we have chosen a generator for a column, other entries of that column shall be calculated based on equilibrium conditions. Therefore, all columns of the B_1 matrix can be calculated sequentially. The problem is to find the generators which can lead to a sparse B_1 matrix.

To select generators such that leads to sparse null vectors, the modified CSS algorithm is applied. For each sequence of selected generators, $\gamma(S)$ systems of linear equations shall be solved simultaneously; such that the sparse solutions with highest number of zeros are found for each system. For this purpose, linear mathematical modeling is proposed.

3. Modified CSS Algorithm for Discrete Binary Variables

Charged System Search (CSS) algorithm is a population based meta-heuristic algorithm. This algorithm is based on laws from electrostatics of physics and Newtonian mechanics. In the CSS algorithm, the movement of CPs is tracked in a continuous domain, i.e. CPs move in